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MANY WAYS TO DESIGN AN EXPERIMENT

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ABSTRACT

After several statistical consulting sessions, it appeared that the statistical consultant's understanding was sufficient to start considering offering possible experiment designs for a forthcoming experiment. Since it was not clear whether all available space and material was to be utilized, experiment designs for both situations are presented. This problem indicates that there may be several ways of conducting an experiment. When the consultee has the various plans, he/she is in a position to make a judgment that best fits the situation.

Keywords: Incomplete block experiment design, resolvable row-column experiment design, Gendex toolkit.

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INTRODUCTION

During the course of statistical consulting, the statistical consultant may have trouble obtaining the desired information from the consultee. One way of having a meeting of the minds is to offer the consultee several designs and/or analyses based upon the statistical consultant's understanding of the problem. Upon discussing the various alternatives, the problem is clearly understood by both parties. Such was the case with problem discussed below. Our understanding of the problems and circumstances surrounding the proposed experiment is used as the basis for the various experiment designs presented below.

Federer (1984) presented the following of his axioms for designing an experiment:

Axiom III: Design for the experiment, do not experiment for the design.

An experiment design should be selected which meets the requirements and desires of the experimenter. This person should not be required to change what he/she wishes to do merely to use a preconceived or catalogued experiment design. That is, an experiment design that uses the material and space available should be constructed. Often the constructed design will be unique to the particular experiment. Designs I and II below make the experimenter "experiment for the design" whereas designs III to VI make use of all available space, i. e., "design for the experiment".

A 1-resolvable design means that there are complete blocks, i. e., all treatments appear in the blocks and each treatment occurs once. A 2-resolvable design has each treatment occurring twice in each of the complete blocks. A non-resolvable design means that the incomplete blocks are not placed in complete blocks but they are randomly scattered over the entire experimental space. A completely randomized design is a non-resolvable design with no incomplete blocks.

THE PROBLEM

An experimenter wishes to conduct a greenhouse experiment. He has been allotted one greenhouse bench that will hold 192 pots. He plans to plant and harvest plants from the 192 pots at four different times. He is interested in comparing $v = 104$ treatments. Whether or not a factorial arrangement of factors is involved was not determined. Therefore, experiment designs are based on 104 treatments without structure. The experimenter is concerned about differences in the response variable for the four time replicates. He is also concerned about variation within benches inside the greenhouse for each of the four time periods. The experimenter states that the bench may be divided into 6 blocks with 32 pots per block. He had planned to rotate the pots in a randomized fashion during the growing period of the plants. This rotation of pots is unnecessary for the following experiment designs II to VI.

In the experiment designs presented below, v is the number of entries (treatments), k is the incomplete block size of k of the v treatments, and r is the number of occurrences (replicates) of a treatment in the experiment.

EXPERIMENT DESIGN I

Using the experimenter's blocking of the bench, i.e. $b = 6$, we note that 6 is not a multiple of 104. If four treatments are duplicates or if 4 additional treatments are added, then $v = 108$ and an incomplete block size of $k = 18$ could be used. Using the Gendex toolkit (See Federer *et al.*, 2000a), an optimal or near-optimal incomplete block experiment design for $v = 108$, $k = 18$, and $r = 4$ in complete blocks (1-resolvable) may be obtained by using either the BIB or the ALPHA module. However, there are 6(14) = 84 places for pots still available on the bench.

EXPERIMENT DESIGN II

If the experimenter does not wish to rearrange the pots in a randomized fashion during the course of the growing season in order to achieve relative homogeneity of environments, he could use a smaller incomplete block. Since 2, 4, and 8 are multiples of

104, the incomplete block size may be k = 2, 4, 8, or 13. If k = 4 is selected, the two arrangements of the four pots in an incomplete block could be

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 _________        _____
|O O O O |       |O O|
                 |O O|
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It would appear that the latter arrangement would make the variation within an incomplete block smaller than the first one. There would be no need to rearrange the pots if an incomplete block of size k = 2 or k = 4 were used. The "edge" effect will be considered later under COMMENTS. This experiment design leaves 88 places for pots unused each time (replicate) the experiment is conducted.

To obtain a randomized form of an experiment design for v = 104, k = 4 (or 2), and r = 4, use the Gendex toolkit, either the BIB module and select a 1-resolvable design or the ALPHA module.

EXPERIMENT DESIGN III

There are 4(192) = 768 places for pots available over the four time periods. 768/104 = 7 + a remainder of 40. Using a non-resolvable plan (i.e., no complete blocks), use the Gendex toolkit to obtain a randomized plan for v = 104, k = 4, and r = 7. Such a design would leave only 40 unused places in time 4. The 4(26) = 104 incomplete blocks would be randomly scattered over the greenhouse bench and over the four time periods. The forty unfilled spaces could be filled with incomplete blocks using treatments 1 to 40 to fill the ten incomplete blocks needed to fill all the space for each time period. Either the BIB or the ALPHA module could be used to obtain a randomized plan for the experiment.

EXPERIMENT DESIGN IV

In order to use a plan that utilizes all the places for pots over the four time periods, consider dividing the 104 treatments into two sets, one of size v = 86 and one of size 18. For the v = 86 treatments in incomplete block sizes of k = 4, use the Gendex toolkit to obtain a randomized plan for v = 86, k = 4, and r = 8. Select a 2-resolvable design, i.e., each of the 86 treatments would appear twice on the bench in any one time period. For the second set of 18 treatments, we need to duplicate or add two treatments so that v = 20. The added treatments should be two treatments from the 86 treatments so that there is link between the two experiments. Use Gendex to obtain a 1-resolvable randomized plan for v = 20, k = 4, and r = 4. This plan would utilize all the places for the pots over the four time periods. The incomplete blocks of the two experiments would be randomly intermingled so that their error variation could be considered to be the same.

EXPERIMENT DESIGN V

Another plan for utilizing all the places for pots over the four time periods is the following. Number treatments from 1 to 104. Then for treatment numbers 105 to 192, use treatments 1 to 88 (or any 88 treatments). Then use Gendex to obtain a 1-resolvable randomized plan for v = 192, k = 4 (or any multiple of 192 such as 2, 3, 6, or 8), r = 4. This plan utilizes all the places for pots over the four time periods. 88 of the treatments will be replicated 8 times each and 104 - 88 = 16 will be replicated four times.

The plan for v = 192, k = 4, and r = 7 as obtained from the BIB module is given in Appendix A.

EXPERIMENT DESIGN VI

Since the pots are placed in a row-column arrangement, a resolvable row-column experiment design could be considered. It appears that the bench has eight rows by 24 columns of pots. An alternative experiment design to V is to use v = 192, k = 8, b = 4(24) = 96, and r = 4. Use may be made of the BIB and the RRC Modules of the Gendex toolkit to obtain the randomized plan for the experiment design (See Federer *et al.*, 2000b).

The randomized plan for v = 192 arranged in k = 8 rows and b = 24 columns in each of the four time periods is given in Appendix B. The ALPHA module was used to create an incomplete block experiment design as the input design and then the RRC module was used to construct the plan given in Appendix B.

COMMENTS

When constructing the data file for experiment designs V and VI, the treatment numbers are entered as 1 to 104 and not as 1 to 192. The number 192 was used merely to construct the incomplete block design.

The incomplete block effects are considered to be random effects as are the time effects. The SAS/MIXED procedure would be used for the statistical analysis. For designs I to V, the edge effect can be entered in the data file as a 1 if the pot is on the edge and 0 if not. This is a fixed effect and can be used as either a class variable or as a covariate. This is because the entry for edge is either a 0 or a 1. The row and column analysis takes care of the edge effect for experiment design VI.

Although the experiment designs presented are mostly for incomplete block sizes of k = 4, the experimenter should seriously consider using incomplete blocks of size k = 2. This block size will control heterogeneity for most of the variation in the experiment even to the elimination of the consideration of the edge effect. Federer and Speed (1987) have demonstrated that with recovery of interblock information (mixed effects model), the efficiency factor for such small block sizes approaches 100%. Of course this is not true if a fixed effects analysis is used as in this case the intrablock efficiency factor for block sizes of k = 2 approaches 50% as v becomes large.

When using Gendex to construct randomized plans, it may be a good idea to insert a number when asked about number of tries. For example, Using the BIB module, v = 200, k = 10, and r = 20, the time required to obtain the plan when no number was entered for number of tries varied between one and three hours. Perhaps 10 tries would be sufficient to get a plan that was more than 99% efficient. If a plan isn't obtained in the desires time, stop the program and reduce the number of tries. A design that is 98% or 99% efficient should be suitable for most purposes.

Since the experiment is to be conducted over four time periods, it is suggested that the data be analyzed at the end of two time periods and again at the end of three time periods. The results of the experiment may become obvious after two or after three time periods and the experiment may be stopped at this point.

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Appendix A

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BIB 2.1: Construct incomplete block designs of size (v,k,r)
(C) 2001 Design Computing (URL:
http://designcomputing.hypermart.net/gendex)

Note: Incomplete block design for v=192, k=4, r=4 and b=192.

try #           1
seed            1016823860520
# of iterations 118
E               0.6817
E/U             0.9925
concurrences    0(17184) 1(1152)

Plan (Blocks are rows):
```

127	176	2	38
10	53	18	187
150	96	63	68
61	123	106	172
4	139	152	43
184	35	189	32
136	82	14	177
121	159	140	103
169	74	110	93
65	111	39	120
130	51	42	133
81	36	163	137
134	64	128	115
16	26	178	180
0	6	138	160
100	15	79	182
86	37	175	88
56	145	161	155
20	50	9	168
153	12	116	107
92	147	174	59
157	146	22	171
60	45	25	148
41	151	158	87
122	113	69	99
70	131	125	126
162	46	144	143
27	57	149	190
78	73	109	11
28	5	85	112
34	76	129	102
31	75	124	164
3	185	188	166
8	54	52	44
156	165	97	7
24	62	48	23
89	58	98	135
49	91	154	95
47	30	132	66
108	141	186	90
29	167	67	55
72	170	19	191
105	94	71	142
21	13	104	17
179	117	181	1
173	118	114	183
84	119	80	40
83	77	101	33

164	142	150	56
10	65	70	43
129	131	147	42
50	87	69	5
76	80	146	39
178	145	20	98
89	170	92	120
114	11	6	37

24	46	22	185
188	113	49	112
181	73	154	152
149	155	140	96
118	31	93	66
54	19	14	60
180	107	177	68
84	7	121	102
79	38	158	141
187	97	41	189
156	0	166	91
64	101	109	15
28	133	110	151
88	139	176	100
108	117	36	167
169	94	125	128
61	153	130	104
35	59	162	134
179	127	132	136
171	72	86	99
144	57	34	175
18	138	30	1
143	105	148	186
4	182	90	67
62	33	172	173
26	106	122	8
17	58	74	53
40	190	44	116
157	95	115	191
165	75	27	23
77	16	21	163
168	119	135	45
51	159	48	126
71	2	29	81
124	85	3	161
137	78	9	83
111	82	13	52
184	63	55	25
174	12	47	103
32	160	123	183

22	6	135	110
52	112	81	129
20	88	116	146
166	102	163	168
176	133	95	50
155	185	122	55
140	29	10	92
13	161	187	91
115	182	183	27
144	2	150	188
90	16	93	123
190	1	63	59
142	174	26	109
84	65	172	151
126	67	9	30
175	12	64	3

47	148	51	69
101	34	164	49
157	78	111	131
53	145	14	128
15	44	167	177
66	99	82	77
36	4	138	75
154	33	125	8
152	105	7	21
80	147	156	127
71	39	136	72
117	68	113	23
97	89	179	96
104	41	143	114
121	83	32	170
171	56	61	189
162	42	160	180
165	159	38	98
134	58	57	124
54	103	18	5
87	186	120	17
86	35	94	181
73	0	62	45
100	173	85	107
79	76	130	169
11	19	48	43
24	108	191	118
132	184	28	137
139	40	106	141
37	153	25	70
31	178	119	158
60	149	74	46

66	8	58	56
124	181	191	106
170	113	76	114
93	153	115	127
31	100	131	149
96	85	80	51
121	4	190	3
116	6	65	128
2	189	107	45
134	117	84	5
97	62	186	177
59	187	112	38
0	17	132	155
148	179	158	10
108	144	43	145
25	174	102	33
142	40	32	67
73	178	18	81
188	26	159	88
146	64	89	138
160	82	94	165
118	79	12	83
44	35	164	140
95	23	147	74

105	157	166	130
48	141	55	7
183	168	136	109
139	41	72	1
78	173	34	156
11	71	87	16
169	9	57	180
24	63	86	104
182	98	91	54
21	184	129	50
122	150	70	137
99	176	125	123
69	172	46	52
185	47	61	68
20	151	162	152
15	30	13	22
126	171	53	163
77	75	110	120
119	19	28	175
60	49	42	29
39	103	90	154
37	161	167	133
92	143	14	27
101	111	135	36

Note: BIB used 16.81 seconds.
Note: this software is licensed to AV Biometrics.

Appendix B:

RRC 2.1: Construct a resolvable row-column design of size (r,k,s)
(C) 2001 Design Computing (URL:
http://designcomputing.hypermart.net/gendex)

Note: Resolvable row-column design for v=192, r=4, k=24 and s=8.

try #	49
seed	-148391148
# of iterations	264
f	4274.44
concurrences	0(8641) 1(8001) 2(1566) 3(125) 4(3)
E(column)	0.9512
E(row)	0.8466
E	0.8068
E/U	0.9885

Plan:

174	116	4	59	142	12	122	18
69	99	105	13	63	162	42	124
25	179	29	112	167	23	176	94
118	17	20	100	6	76	9	157
83	117	27	156	155	41	46	107
175	148	143	165	168	11	172	56
149	70	32	138	141	166	0	125

36	145	84	38	67	110	31	57
28	144	96	147	146	152	49	68
80	130	169	22	173	177	108	79
182	66	134	2	188	74	53	98
164	78	185	47	171	187	34	60
48	19	160	55	58	151	139	21
91	77	43	153	90	92	85	136
135	7	35	120	52	40	16	3
123	50	154	163	64	5	45	71
184	129	133	44	81	82	51	62
114	15	170	10	132	24	95	159
33	104	30	97	86	73	111	106
161	191	103	137	127	189	14	115
109	158	39	183	89	102	131	121
26	180	93	150	8	75	140	72
178	88	190	128	61	87	126	113
119	181	186	101	1	37	65	54

32	165	80	38	82	54	131	107
102	163	146	35	15	76	106	79
101	71	129	52	20	172	87	36
191	9	4	41	84	61	90	135
72	10	149	176	14	58	99	57
74	164	126	27	180	70	112	6
175	13	189	21	7	29	170	51
67	152	118	137	125	117	93	154
182	19	111	59	181	0	81	92
120	77	145	47	49	83	127	105
157	122	143	63	89	40	33	22
177	65	94	151	43	161	100	142
190	110	114	104	8	147	46	115
160	130	26	68	56	34	186	23
64	11	24	169	183	42	174	88
86	3	75	141	53	50	48	85
187	103	166	123	18	173	31	148
69	128	39	140	179	44	60	139
119	124	153	113	185	158	16	155
95	138	78	98	178	150	5	167
25	12	45	62	168	91	66	30
184	2	162	1	132	116	96	121
144	109	97	133	17	188	108	37
55	171	28	136	156	73	134	159

154	53	165	177	158	149	90	68
29	104	127	15	92	26	55	65
166	52	42	19	157	85	56	27
86	186	28	183	124	117	108	38
100	51	152	50	160	73	14	60
91	96	130	125	75	64	35	109
171	13	111	57	190	82	123	94
129	174	140	188	151	147	32	159
180	133	10	119	134	167	80	106
139	173	25	189	132	84	97	71
169	95	141	126	148	76	44	33
107	153	122	47	161	74	1	7
182	168	17	150	131	103	105	113
110	93	101	30	83	78	23	58

40	48	185	136	88	2	39	137
121	164	43	63	144	172	16	145
191	118	155	49	138	87	62	162
34	156	72	179	135	81	20	12
77	102	112	116	45	37	178	41
99	89	163	54	36	114	187	9
61	115	31	6	59	175	24	66
79	70	4	3	176	69	8	5
143	128	98	184	170	18	67	0
181	120	21	46	146	22	142	11

119	144	174	29	168	22	14	41
157	12	149	104	132	16	129	54
78	112	184	107	35	84	65	118
46	79	97	152	92	32	89	58
96	191	30	113	13	176	117	11
88	95	9	180	28	3	82	127
108	63	21	128	166	20	182	147
61	49	123	19	72	74	185	100
189	190	155	99	53	44	47	18
60	86	161	4	36	55	76	130
151	136	181	71	6	93	135	69
154	56	109	169	87	111	116	124
105	67	75	34	122	66	43	133
103	1	156	57	177	77	42	114
81	140	160	37	121	91	138	7
52	139	24	2	80	148	68	150
70	187	85	106	188	183	110	25
165	51	145	125	23	27	173	134
102	83	167	31	137	170	50	90
98	15	175	142	45	162	186	164
94	59	126	48	73	101	8	143
33	158	146	64	38	115	10	0
153	131	40	159	179	163	178	141
26	5	171	62	120	17	172	39

Note: RRC used 56.29 seconds.
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